So
$$\boxed{\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}(m_1+m_2)\ell_1^2\dot\theta_1^2 + \tfrac{1}{2}m_2\ell_2^2\dot\theta_2^2 \\ & + m_2\ell_1\ell_2\dot\theta_1\dot\theta_2\cos(\theta_1-\theta_2) + (m_1+m_2)g\ell_1\cos\theta_1 \\ & + m_2 g\ell_2\cos\theta_2\end{aligned}}$$

(I have dropped constants like $(m_1+m_2)g\ell_1$ because you can add any constant to a potential energy).

$$\frac{\partial\mathcal{L}}{\partial\dot\theta_1} = (m_1+m_2)\ell_1^2\dot\theta_1 + m_2\ell_1\ell_2\dot\theta_2\cos(\theta_1-\theta_2)$$

$$\frac{\partial\mathcal{L}}{\partial\theta_1} = -m_2\ell_1\ell_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2) - (m_1+m_2)g\ell_1\sin\theta_1$$

2 terms $\pm m_2\ell_1\ell_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2)$ cancel

θ_1 eqn:
$$\boxed{\begin{aligned}&(m_1+m_2)\ell_1^2\ddot\theta_1 + m_2\ell_1\ell_2\ddot\theta_2\cos(\theta_1-\theta_2) \\ &+ m_2\ell_1\ell_2\dot\theta_2^2\sin(\theta_1-\theta_2) + (m_1+m_2)g\ell_1\sin\theta_1 = 0\end{aligned}}$$

$$\frac{\partial\mathcal{L}}{\partial\dot\theta_2} = m_2\ell_2^2\dot\theta_2 + m_2\ell_1\ell_2\dot\theta_1\cos(\theta_1-\theta_2)$$

$$\frac{\partial\mathcal{L}}{\partial\theta_2} = +m_2\ell_1\ell_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2) - m_2 g\ell_2\sin\theta_2$$

θ_2 eqn:
$$\boxed{\begin{aligned}&m_2\ell_2^2\ddot\theta_2 + m_2\ell_1\ell_2\ddot\theta_1\cos(\theta_1-\theta_2) \\ &+ m_2\ell_1\ell_2\dot\theta_1^2\sin(\theta_1-\theta_2) + m_2 g\ell_2\sin\theta_2 = 0\end{aligned}}$$