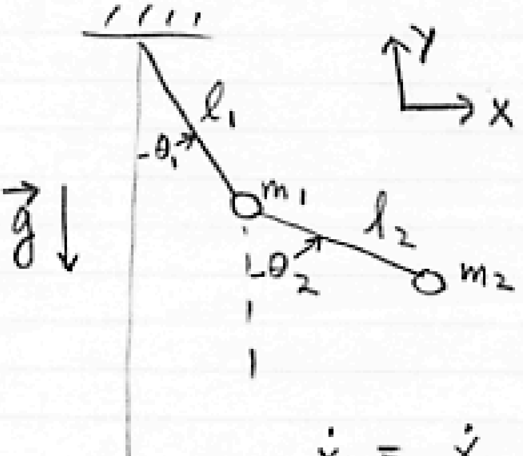
<u>Goldstein 1.22</u> The double pendulum. Pick θ_1, θ_2 as coords.



$$x_1 = l\sin\theta_1 \qquad y_1 = -l\cos\theta_1$$

$$x_2 = x_1 + l_2\sin\theta_2$$

$$y_2 = y_1 - l_1\cos\theta_2$$

$$\dot{x}_1 = l\dot{\theta}_1\cos\theta_1 \qquad \dot{y}_1 = l_1\dot{\theta}_1\sin\theta_1$$

$$\dot{x}_2 = \dot{x}_1 + l_2\dot{\theta}_2\cos\theta_2$$

$$\dot{y}_2 = \dot{y}_1 + l_2\dot{\theta}_2\sin\theta_2$$

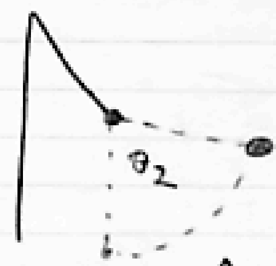
Hence

$$T = \Sigma \tfrac{1}{2} m_i \vec{v}_i^{\,2}$$

$$= \tfrac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 \left(l_1^2 \dot{\theta}_1^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos\theta_1 \cos\theta_2 + l_2^2 \dot{\theta}_2^2 [\cos^2 + \sin^2] + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_1 \sin\theta_2 \right).$$

$$T = \tfrac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 \left(l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 \right) + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$V = \underbrace{(m_1 + m_2) g l_1 (1 - \cos\theta_1)} + \underbrace{m_2 g l_2 (1 - \cos\theta_2)}$$



Add potential energy of this configuration to that of this one. this trick involves path independence of the def. of V

$$V(\vec{r}) = -\int_{r_0}^{\vec{r}} \nabla F \cdot d\underset{\sim}{l}.$$