Notice that Newton's 2nd Law is <u>not</u> invariant under point transformations. For force-free motion:

$$m\frac{d^2x^i}{dt^2} = 0 \qquad ; \quad \text{let} \quad x^i = x^i(s_1, s_2 \cdots s_n)$$

$$0 = \frac{d^2x_i}{dt^2} = \frac{d}{dt}\left(\sum_{k=1}^{n} \frac{\partial x_i}{\partial s_k}\dot{s}_k\right)$$

$$= \sum_{k=1}^{n}\left(\sum_{j=1}^{n} \frac{\partial^2 x_i}{\partial s_j \partial s_k}\dot{s}_j\dot{s}_k + \frac{\partial x_i}{\partial s_k}\ddot{s}_k\right)$$

$$= \sum_{k=1}^{n} \frac{\partial x_i}{\partial s_k}\left[\frac{d^2 s_k}{dt^2} + \sum_{\substack{j=1\\m,n}}^{n} \frac{\partial s_k}{\partial x_j}\frac{\partial^2 x_j}{\partial s_m \partial s_n}\dot{s}_m\dot{s}_n\right]$$

so that:

$$\frac{d^2 s_k}{dt^2} = -\sum_{k,m,n}\left(\frac{\partial^2 x_j}{\partial s_m \partial s_n}\frac{\partial s_k}{\partial x_j}\right)\frac{ds_m}{dt}\frac{ds_n}{dt} \qquad \neq 0$$

The term

$$\Gamma^k_{mn} = \frac{\partial^2 x_j}{\partial s_m \partial s_n}\frac{\partial s_k}{\partial x_j}$$

is related to the Christoffel symbols of General Relativity.