

$$d) \quad \langle T_{cm} \rangle = -\frac{1}{2} \langle U \rangle + W = \langle T_{cm} \rangle + \langle U \rangle = \langle T_{cm} \rangle - 2\langle T_{cm} \rangle$$

$$= -\langle T_{cm} \rangle = -\frac{3}{2} N k \tau$$

$$\therefore \boxed{\tau = -\frac{2W}{3Nk}}$$

As W decreases, $-W$ increases making τ larger.

By part c) when τ increases R decreases.

e) By the virial theorem $\langle T_{cm} \rangle = -\frac{1}{2} \langle U \rangle \neq 0 \Rightarrow$ molecules must radiate. Now apply the arguments of part (d) above.

f) From part (a),

$$U = -\frac{3}{5} \frac{M^2 G}{R} = -\frac{3}{5} \frac{G}{R} \left(\frac{4\pi}{3} \rho R^3 \right)$$

$$\Rightarrow \boxed{U = -\frac{16\pi^2}{15} G \rho^2 R^5}$$

$$2T_{cm} = 3Nk\tau$$

$$N = \frac{M}{m} = \frac{\frac{4}{3}\pi R^3 \rho}{m} \Rightarrow$$

$$\boxed{2T_{cm} = \frac{4\pi k \tau \rho R^3}{m}}$$