

The computation of  $U$  is now immediate:

$$U = \frac{1}{2} \int_{\text{sphere}} d^3 \vec{r} \rho_0 \left[ \frac{1}{2} \frac{4\pi}{3} G \rho_0 r^2 - \frac{3}{2} \frac{4\pi}{3} G \rho_0 R^2 \right]$$

$$= \frac{1}{2} 4\pi \frac{1}{2} \frac{4\pi}{3} G \rho_0^2 \int_0^R r^2 dr [r^2 - 3R^2]$$

$$= \frac{1}{2} 4\pi \frac{1}{2} \frac{4\pi}{3} G \rho_0^2 \left\{ \frac{r^5}{5} - 3 \frac{R^2 r^3}{3} \right\} \Big|_0^R$$

$$= \frac{1}{2} 4\pi \frac{1}{2} \frac{4\pi}{3} G \rho_0^2 \left( -\frac{4}{5} \right) R^5$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot 3 \left\{ \left( \frac{4\pi}{3} \right)^2 \rho_0^2 R^6 \right\} \frac{G}{R} \left( -\frac{4}{5} \right)$$

$$U = -\frac{3}{5} \frac{M^2 G}{R}$$

b)  $\langle 2T_{cm} \rangle = -\langle U \rangle$ . Argument the same as in problem DCM 1.18, part a

c)  $\langle 2T_{cm} \rangle = 3Nk\tau$  and part (a)  $\Rightarrow$

$$\tau = \frac{M^2 G}{5RN} = \frac{m M G}{5kR} \quad \text{with } m = \frac{M}{N}$$