$$\Rightarrow \hat{e}_r \cdot \vec{g}(\vec{r})\, 4\pi r^2 = -4\pi G \rho_0 \frac{4}{3}\pi r^3 \qquad r \le R$$

$$= -4\pi G \rho_0 \frac{4}{3}\pi R^3 \qquad r \ge R$$

$$\Rightarrow \vec{g}(\vec{r}) = -\hat{e}_r \frac{4\pi}{3} G\rho_0 r \qquad r \le R$$

$$\vec{g}(\vec{r}) = -\hat{e}_r \frac{4\pi}{3} G\rho_0 \frac{R^3}{r^2} \qquad r \ge R$$

$$\Rightarrow \phi(r) = \frac{1}{2}\frac{4\pi}{3} G\rho_0 r^2 + \text{const} \qquad r \le R$$

$$\phi(r) = -\frac{4\pi}{3} G\rho_0 R^3 \frac{1}{r} + \text{const}' \qquad r \ge R$$

Also, $\text{const}' = 0$ since $\phi(\infty) = 0$.

$\therefore$ $\text{const} = -\frac{4\pi}{3} G\rho_0 R^2 - \frac{1}{2}\frac{4\pi}{3} G\rho_0 R^2 = -\frac{3}{2}\frac{4\pi}{3} G\rho_0 R^2$

since continuity $\Rightarrow \phi(R+\epsilon) = \phi(R-\epsilon)$

$\therefore$

$$\phi(r) = \frac{1}{2}\frac{4\pi}{3} G\rho_0 r^2 - \frac{3}{2}\frac{4\pi}{3} G\rho_0 R^2 \qquad r \le R$$

$$= -\frac{4\pi}{3} G\rho_0 \frac{R^3}{r} \qquad r \ge R$$