a) $\ddot{S} = (2T_{cm} + U) \Rightarrow \frac{1}{T}\int_0^T dt\, \ddot{S} = \frac{1}{T}\int_0^T (2T_{cm} + U)\,dt$

$\Rightarrow \langle 2T_{cm} + U \rangle = \lim_{T\to\infty} \frac{1}{T}\left(\dot{S}(T) - \dot{S}(0)\right) = 0$

$\Rightarrow \boxed{\langle 2T_{cm} \rangle = -\langle U \rangle}$

b) $2T_{cm} = \sum m_i \vec{v}_i^{\,\prime 2} = m \sum \vec{v}_i^{\,\prime 2} = m N v^{*2}$ or

$2T_{cm} = M v^{*2} \Rightarrow \boxed{\langle 2T_{cm} \rangle = M v^{*2}}$

$U = -\frac{1}{2} G \sum_{i \neq j} \frac{m_i m_j}{\| \vec{r}_i - \vec{r}_j \|} = -m^2 G \frac{1}{2} \sum_{i,j} \frac{m_i m_j}{\| \vec{r}_i - \vec{r}_j \|}$

$= -m^2 G N^2 / r^* = -M^2 G / r^*$

$U = -\frac{M^2 G}{r^*} \Rightarrow \boxed{\langle U \rangle = -\frac{M^2 G}{r^*}}$ Combine

these results with part a $\Rightarrow M v^{*2} = \frac{M^2 G}{r^*}$

or $\boxed{M = G^{-1} r^* v^{*2}}$