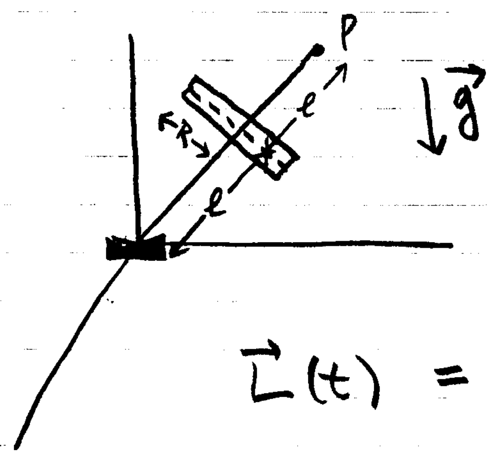
DCM1.16



$\vec{r}_p$ = vector from O to P

$$\vec{r}_p(t) = 2l\cos\theta\,\hat{e}_z + 2l\sin\theta\left(\hat{e}_x\cos\Omega t + \hat{e}_y\sin\Omega t\right)$$

Since $\omega \gg \Omega$ the angular momentum is

$$\vec{L}(t) = \frac{\vec{r}_p}{2l}\,\omega I \qquad I = \text{flywheel moment of inertia} = \frac{MR^2}{2}$$

$$\frac{d\vec{L}}{dt} = \omega\Omega I\sin\theta\left(-\hat{e}_x\sin\Omega t + \hat{e}_y\cos\Omega t\right)$$

But torque $\vec{N} = \left(\frac{\vec{r}_p}{2}\right)\times\vec{F}$ w/ $\vec{F} = -Mg\hat{e}_z$

$$\vec{N} = Mgl\sin\theta\left(\hat{e}_y\cos\Omega t - \hat{e}_x\sin\Omega t\right)$$

Thus $\frac{d\vec{L}}{dt} = \vec{N}$ gives $\omega\Omega I = Mgl$

or, solving for Ω, $\boxed{\Omega = \frac{Mgl}{\omega I} = \frac{2gl}{\omega R^2}}$

$$M\ddot{\vec{r}}_{cm} = \vec{F}_{total} = -Mg\hat{e}_z + \vec{F}_{cup\to top}$$

$$\vec{F}_{cup\to top} = \tfrac{1}{2}M\ddot{\vec{r}}_p + Mg\hat{e}_z$$

$$\boxed{\vec{F}_{cup\to top} = -Ml\Omega^2\sin\theta\left(\hat{e}_x\cos\Omega t + \hat{e}_y\sin\Omega t\right) + Mg\hat{e}_z}$$

with Ω given above.