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See Symon, Page 173

cles in the system, these must be taken into account.

A typical problem in which the linear momentum theorem is applicable is the conveyor belt problem. Material is dropped continuously from a hopper onto a moving belt, and it is required to find the force F required to keep the belt moving at constant velocity v (Fig. 4-2). Let the rate at which mass is dropped on the belt be dm/dt. If m is the mass of material on the belt, and M is the mass of the belt (which really does not figure in the problem), the total momentum of the system, belt plus material on the belt and in the hopper, is

$$P = (m + M)v. \qquad (4\text{-}50)$$

We assume that the hopper is at rest; otherwise the momentum of the hopper and its contents must be included in Eq. (4-50). The linear momentum theorem requires that

$$F = \frac{dP}{dt} = v\frac{dm}{dt}. \qquad (4\text{-}51)$$

This gives the force applied to the belt. The power supplied by the force is

$$Fv = v^2\frac{dm}{dt} = \frac{d}{dt}(mv^2) = \frac{d}{dt}[(m+M)v^2]. \qquad (4\text{-}52)$$

This is twice the rate at which the kinetic energy is increasing, so that the

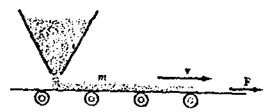


FIG. 4-2. A conveyor belt.

conservation theorem of mechanical energy (4-40) does not apply here. Where is the excess half of the power going?

a) $F = v\dfrac{dm}{dt}$

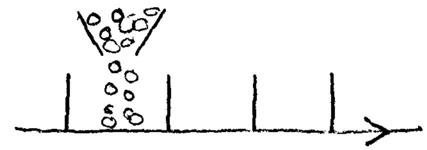
b) $P = Fv = v^2\dfrac{dm}{dt}$

c) $\dfrac{d}{dt}(KE) = \dfrac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2\dfrac{dm}{dt}$

d) $\therefore \dfrac{d}{dt}(KE) \neq P!$, in fact

$$P = \frac{d}{dt}(KE) + \frac{1}{2}v^2\frac{dm}{dt}$$

Where does energy go! Imagine the belt had vanes as shown below:



Then the sand, which is at rest, is struck by a vane moving with velocity v. Let the vane have mass $M >>$ mass of sand grains. Then, if the collision is <u>elastic</u>, the grain must move with velocity <u>$2v$</u>! But, the problem has built in the requirement that the sand <u>must</u> move with velocity v, $\therefore$ the collision is <u>completely</u> inelastic, $\therefore$ the sand gets hot! Let us compute the energy loss when mass "dm" of sand collides with the vane. Let $v_{\substack{b\\a}} = v\left(\substack{\text{before}\\\text{after}}\right)$ collision.

Momentum conservation $\Longrightarrow$ $\boxed{Mv_b = Mv_a + dm\,v_a}$

Let $E_{\substack{b\\a}}$ = Energy $\left(\substack{\text{before}\\\text{after}}\right)$

$E_b = \frac{1}{2}Mv_b^2$, $E_a = \frac{1}{2}Mv_a^2 + \frac{1}{2}dm\,v_a^2$. Solving $\Rightarrow$

$$E_b - E_a = \left[\frac{1}{2}M\left(\frac{M+dm}{M}\right)^2 - \frac{1}{2}M - \frac{1}{2}dm\right]v_a^2 \Rightarrow$$

$$\Delta E = \frac{1}{2}dm\,v_a^2 + O(dm^2) \Rightarrow$$

$\boxed{dE/dt = \frac{1}{2}v^2\dfrac{dm}{dt}}$ which is just the energy to be accounted for!