DLN 2.4.4 $\quad y^{n+1} = y^n + h \sum_{j=0}^{N} a_j \nabla^j f^{n+1}$

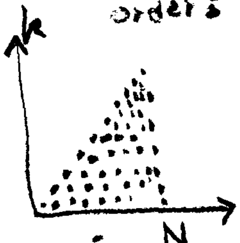
But $\quad \nabla^j f^{n+1} = \sum_{k=0}^{j} (-1)^k \binom{j}{k} f^{n+1-k}$

$$y^{n+1} = y^n + h \sum_{j=0}^{N} a_j \sum_{k=0}^{j} (-1)^k \binom{j}{k} f^{n+1-k}$$

$$= y^n + h \sum_{k=0}^{N} (-1)^k f^{n+1-k} \sum_{j=k}^{N} \binom{j}{k} a_j$$

I.e. $\quad \tilde{a}_k^N = (-1)^k \sum_{j=k}^{N} \binom{j}{k} a_j$

Interchange summation order:



$$\sum_{j=0}^{N} \sum_{k=0}^{j} = \sum_{k=0}^{N} \sum_{j=k}^{N}$$

Clearly since no earlier f^* than f^{n+1-N} is accessed by this formula, we can write

$$\tilde{a}_N^N = (-1)^N a_N, \quad \tilde{a}_k^N = 0 \quad k > N$$

Also $\quad \tilde{a}_k^{N+1} = (-1)^k \sum_{j=k}^{N+1} \binom{j}{k} a_j$

$$= (-1)^k \binom{N+1}{k} a_{N+1} + \tilde{a}_k^N .$$

The $\tilde{b}$'s we can do similarly:

$$y^{n+1} = y^n + h \sum_{j=0}^{N} b_j \nabla^j f^n$$

$$= y^n + h \sum_{j=0}^{N} b_j \sum_{k=0}^{j} \binom{j}{k} f^{n-k} (-1)^k$$