DLN 2.4.3 contd.

Clearly $F(z) = (1-z)\,G(z)$ so

$$\sum_{k=0}^{\infty} a_k z^k = (1-z)\sum_{k=0}^{\infty} b_k z^k$$

$$= \sum_{k=0}^{\infty} b_k z^k - \sum_{k=0}^{\infty} b_k z^{k+1}$$

$$= \sum_{k=0}^{\infty} b_k z^k - \sum_{\ell=1}^{\infty} b_{\ell-1} z^{\ell}$$

Set $b_{-1} = 0$ for convention & we can wri

$$\sum_{k=0}^{\infty} a_k z^k = \sum_{k=0}^{\infty} (b_k - b_{k-1}) z^k \quad \text{or}$$

$$\boxed{a_k = b_k - b_{k-1}}$$

$$b_0 = a_0 = 1$$

$$b_1 = a_1 + b_0 = -\tfrac{1}{2} + 1 = \tfrac{1}{2},$$

$$b_2 = a_2 + b_1 = -\tfrac{1}{12} + \tfrac{1}{2} = \tfrac{5}{12},$$

$$b_3 = a_3 + b_2 = -\tfrac{1}{24} + \tfrac{5}{12} = \tfrac{9}{24},$$

$$b_4 = a_4 + b_3 = -\tfrac{19}{720} + \tfrac{9}{24} = \tfrac{251}{720}.$$