DLN 2.4.3

$$F(z) = -\frac{z}{\log(1-z)} = \sum_{k=0}^{\infty} a_k z^k$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\log(1-x) = -\int_0^x \frac{dt}{1-t} = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots\right)$$

$$\frac{-x}{\log(1-x)} = \frac{x}{x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots}$$

Now just divide out:

$$1 - \frac{x}{2} - \frac{x^2}{12} - \frac{x^3}{24} - \frac{19x^4}{720}$$

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} \;\Big)\; x$$

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + x^5/5$$

$$-\frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}$$

$$-\frac{x^2}{2} - \frac{x^3}{4} - \frac{x^4}{6} - \frac{x^5}{8}$$

$$-\frac{x^3}{12} - \frac{x^4}{12} - \frac{3x^5}{40}$$

$$-\frac{x^3}{12} - \frac{x^4}{24} - \frac{x^5}{36}$$

$$-\frac{x^4}{24} - \frac{34x^5}{720}$$

$$-\frac{x^4}{24} - \frac{x^5}{48}$$

$$-\frac{19x^5}{720}$$

Radius of convergence $|z-1| < 1$ since the log has a singularity at ∂arg $= 0$.

There is another method.

$$F(x) = \frac{1}{1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \cdots}$$

Write $\left(1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} \cdots\right) F(x) = 1$

and equate powers. But this is equivalent to the synthetic division.