D2.4.2 cont.

Similarly, $\nabla f^{n+1} = 6ht^{n+1} - 3h^2$

$$\nabla^2 f^{n+1} = 6h^2, \quad \nabla^k f^{n+1} = 0 \quad \text{for } k \geq 3$$

Equations (4.27) and (4.28) read

$$y^{n+1} = y^n + h \sum_0^N a_k \nabla^k f^{n+1}$$

$$y^{n+1} = y^n + h \sum_0^N b_k \nabla^k f^n$$

Since $\nabla^k f^n = \nabla^k f^{n+1} = 0$ when $k \geq 3$, these formulas should be exact when $N = 2$ for this problem. Therefore we should check the relation

$$y^{n+1} \stackrel{?}{=} y^n + h\left[a_0 f^{n+1} + a_1 \nabla f^{n+1} + a_2 \nabla^2 f^{n+1}\right]$$

$$y^{n+1} \stackrel{?}{=} y^n + h\left[b_0 f^n + b_1 \nabla f^n + b_2 \nabla^2 f^n\right].$$

$a_0 = 1$ $\qquad a_1 = -1/2$ $\qquad a_2 = -1/12$

$b_0 = 1$ $\qquad b_1 = 1/2$ $\qquad b_2 = 5/12$