DLN 2.4.2

Suppose $y(t) = t^3$. Then $f = \dot{y} = 3t^2$

$\therefore\ y^n = (t^n)^3$, $f^n = 3(t^n)^2$. For the predictor and corrector formulas we need $\nabla^k f^n$ and $\nabla^k f^{n+1}$. We find the results

$$\nabla f^n = f^n - f^{n-1} = 3\left[(t^n)^2 - (t^{n-1})^2\right] = 3\left[(t^n)^2 - (t^n - h)^2\right]$$

$$= 3\left\{(t^n)^2 - \left[(t^n)^2 - 2ht^n + h^2\right]\right\}$$

$$\Rightarrow \boxed{\nabla f^n = 6ht^n - 3h^2}$$

$$\nabla^2 f^n = f^n - 2f^{n-1} + f^{n-2} = 3\left[(t^n)^2 - 2(t^{n-1})^2 + (t^{n-2})^2\right]$$

$$= 3\left\{(t^n)^2 - 2(t^n - h)^2 + (t^n - 2h)^2\right\}$$

$$= 3\left\{(t^n)^2 - 2\left[(t^n)^2 - 2ht^n + h^2\right] + \left[(t^n)^2 - 4ht^n + 4h^2\right]\right\}$$

$$= 6h^2$$

Evidently $\boxed{\nabla^k f^n = 0 \quad \text{for } k \geq 3}$