

(4.5) $\nabla 1 = 0$. Easy. $\nabla 1 = 1 - 1 = 0$.

(4.6) Clearly $\nabla t^d = t^d - (t-h)^d$

is a polynomial of degree $d-1$, and in general if $p(t)$ is a polynomial of degree d , $\nabla p(t)$ is a polynomial of degree $d-1$.

So $\nabla^m t^m$ is a polynomial of degree zero, i.e. a constant, and

$$\nabla^{m+1} t^m = \nabla(\text{const}) = \text{const} \cdot \nabla(1) = 0$$

Hence $\nabla^l t^m = \nabla^{l-m-1} \nabla^{m+1} t^m = 0$
if $l > m$.

(4.7) $\nabla t^d = t^d - (t-h)^d$
 $= dh t^{d-1} + \dots$

So claim: $\nabla^p t^d = \frac{d!}{(d-p)!} h^p t^{d-p} + \dots$ for $p \leq d$

For $p=1$ it works as we've just shown
Otherwise assume it works for p &

$$\begin{aligned} \nabla^{p+1} t^d &= \frac{d!}{(d-p)!} h^p \{ (d-p) h t^{d-p-1} \} + \dots \\ &= \frac{d!}{(d-(p+1))!} h^{p+1} t^{d-(p+1)} + \dots \end{aligned}$$

OK. Now evaluate for $p = d = m$

$$\nabla^m t^m = \frac{m!}{0!} h^m + \text{terms which must by now be zero since the highest order term is a const}$$

$\| \nabla^m t^m = m! h^m \|$