DLN
2.4.1

$$(4.4)\quad \nabla^m \vec{y}^{\,n} = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \vec{y}^{\,n-k}$$

For $m=1$ the f'la reads

$$\nabla \vec{y}^{\,n} = \vec{y}^{\,n} - \vec{y}^{\,n-1}, \quad \text{obviously true.}$$

Assume (4.3) is true for $m$ & then try to p've for $m+1$. Take $\nabla$ of both sides

$$\nabla^{m+1} \vec{y}^{\,n} = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \left( \vec{y}^{\,n-k} - \vec{y}^{\,n-k-1} \right).$$

$$= \sum_{k=0}^{m} (-1)^k \binom{m}{k} \vec{y}^{\,n-k} - \sum_{k=1} (-1)^{k-1} \frac{m!}{(k-1)!\,(m-k+1)!} y^{n-k}$$

$K_{new} = k+1$

$$= \sum_{k=0}^{m} (-1)^k \frac{m!}{k!\,(m-k)!} \vec{y}^{\,n-k} + \sum_{k=0}^{m+1} (-1)^k \frac{k\,m!}{k!\,(m+1-k)!} \vec{y}^{\,n-k}$$

the $K=0$ term doesn't contribute.

$$= \sum_{k=0}^{m} (-1)^k \frac{(m+1)!}{k!\,(m+1-k)!} \frac{m+1-k}{m+1} y^{n-k}$$

$$+ \sum_{k=0}^{m+1} (-1)^k \frac{(m+1)!}{k!\,(m+1-k)!} \left(\frac{k}{m+1}\right) \vec{y}^{\,n-k}$$

$$= \sum_{k=0}^{m} (-1)^k \frac{(m+1)!}{k!\,(m+1-k)!} \vec{y}^{\,n-k} + (-1)^{m+1} \frac{(m+1)!}{(m+1)!\,0!} \vec{y}^{\,n-k}$$

$$= \sum_{k=0}^{m} (-1)^k \binom{m+1}{k} \vec{y}^{\,n-k} \qquad !$$

So the induction is completed.