D2.3.1 cont.

$$\frac{d^2\vec{f}^n}{dt^2} = \check{\check{\vec{f}}}^n + 2\frac{dy_k}{dt}\check{\vec{f}}^n_{,k} + \frac{d^2y_k}{dt^2}\vec{f}^n_{,k}$$

$$+ \frac{dy_k}{dt}\frac{dy_l}{dt}\vec{f}^n_{,kl}$$

Thus, Taylor series gives

$$\vec{y}^{n+1}_{true} = \vec{y}^n + h\vec{f}^n + \tfrac{1}{2}h^2\check{\vec{f}}^n + \tfrac{1}{2}h^2 f^n_k \vec{f}^n_{,k}$$

$$+\tfrac{1}{6}h^3\check{\check{\vec{f}}}^n + \tfrac{2}{6}h^3 f^n_k \check{\vec{f}}^n_{,k} + \tfrac{1}{6}h^3\frac{d^2y_k}{dt^2}\vec{f}^n_{,k}$$

$$+\tfrac{1}{6}h^3 f^n_k f^n_l \vec{f}^n_{,kl} + O(h^4)$$

Comparing with (2) we see that the coeff's of $1, h$ & h^2 are equal. The difference is in 3rd order terms and is

$$\vec{c}h^3 + O(h^4) = \vec{y}^{n+1}_{true} - \vec{y}^{n+1}_{approx}$$

w/ $$\vec{c} = \tfrac{1}{6}\check{\check{\vec{f}}} + \tfrac{2}{6}f_k\check{\vec{f}}_{,k} + \tfrac{1}{6}\ddot{y}_k\vec{f}_{,k}$$

$$+\tfrac{1}{6}f_k f_l \vec{f}_{,kl} - \tfrac{1}{4}\check{\check{\vec{f}}} - \tfrac{2}{4}f_k\check{\vec{f}}_{,k}$$

$$-\tfrac{1}{4}f_k f_l \vec{f}_{,kl}$$

Add & Subtract $\tfrac{1}{4}\ddot{y}_k\vec{f}_{,k}$

$$c = -\tfrac{1}{12}\left[\check{\check{\vec{f}}} + 2f_k\check{\vec{f}}_{,k} + \ddot{y}_k\vec{f}_{,k} + f_k f_l \vec{f}_{,kl}\right]$$

$$+\tfrac{1}{4}\ddot{y}_k\vec{f}_{,k}$$

$$\left\| \; \vec{c} = -\tfrac{1}{12}\dddot{\vec{y}} + \tfrac{1}{4}\ddot{y}_k\vec{f}_{,k} \; \right\|$$