

$$\begin{aligned} y_1^{n+1} &= y_1^n + h y_2^n \\ y_2^{n+1} &= y_2^n - h y_1^n + 2nh^2 + 2t^0 h \end{aligned} \quad (†)$$

In linear difference equations, as in linear differential equations, it is easy to show that the general solution is a particular solution of the nonhomogeneous equation plus any solution of the homogeneous equation. The proof is easy. Let  $\vec{y}_p^n$  be a particular solution and  $\vec{z}^n$  an arbitrary solution of the non-homogeneous eqn. Then

$\vec{z}^n - \vec{y}_p^n$  is a solution of the homogeneous equation, by linearity. So letting  $\vec{y}_0^n$  be the homog. soln.,

$$\vec{z}^n = \vec{y}_p^n + \vec{y}_0^n.$$

OK. Now we can find a particular solution with ease (because I already know it)

$$\left. \begin{aligned} y_{1p}^n &= 2nh + 2t^0 \\ y_{2p}^n &= 2 \end{aligned} \right\} (*)$$

Use (†) to find  $\vec{y}_p^{n+1}$

$$y_{1p}^{n+1} = 2nh + 2h + 2t^0 = 2(n+1)h + 2t^0$$

$$y_{2p}^{n+1} = 2 - 2nh^2 - 2t^0 h + 2t^0 h + 2nh^2$$

which checks with (\*) for  $n \rightarrow n+1$ .

Now we note that  $y_1(t) = 2t$  is a particular solution to the exact differential equations also, and hence in  $y_{\text{ex}}(t) - y^n$  the particular solutions drop out.