Note: Can also write as $e^{iT} - e^{N\log(1+\frac{iT}{N})} = e^{iT} - e^{iN(\frac{iT}{N} - (\frac{iT}{N})^2\frac{1}{2}\cdots)}$

$= e^{iT}\{1 - e^{i\frac{T^2}{2N}} + O(\frac{1}{N^2})\}$ which is $\sim \frac{1}{N}$ 3/4

L-DLN 2.2.1 cont

$$e^{iT} - (1+iT/N)^N = 1 + iT - \frac{T^2}{2} - \frac{iT^3}{3!} + \ldots$$
$$-1 - iT + \frac{N^2-N}{2N^2}T - iT^3\frac{N^3-3N^2}{N^3}$$
$$= -\frac{T^2}{2N} + \frac{3iT^3}{N} + \ldots, \sim \frac{f(T)}{N}$$

Similarly $e^{-iT} - (1-iT/N)^N = -\frac{T^2}{2N} \ldots = \frac{f(-T)}{N}$ (by replacing $T \to -T$). Also $f^*(T) = f(-T)$ so

$$|\vec{y}_{ex}(t) - \vec{y}(t)| \leq \frac{(|A|+|B|)|f(T)|}{N} \sim \frac{1}{N}$$

or $\sim h$.

ie. goes to zero linearly with h.

(e) For fixed h the approx. sol. is

$$\vec{y}^n = A(1+ih)^n\vec{a} + B(1-ih)^n\vec{b}$$
$$\vec{y}^{N*}\cdot\vec{y}^N = |A|^2|1+ih|^{2N} + |B|^2|1-ih|^{2N}$$
$$= (|A|^2 + |B^2|)(1+h^2)^N,$$

which $\to \infty$ as $N \to \infty$ for fixed $h \neq 0$.
True solution is oscillatory as $t \to \infty$.

(f). EG#1 : $\ddot{x} + x = 2t$. Again $y_1 = x$, $y_2 = \dot{x}$

Thus $\dot{y}_1 = y_2 = f_1$

$\dot{y}_2 = -y_1 + 2t = f_2$

We make this into a difference eqn. to use the Euler technique: