$\vec{a}$ & $\vec{b}$ are linearly independent, since

$$c_1\vec{a} + c_2\vec{b} = 0 \Rightarrow \begin{pmatrix} c_1 \\ ic_1 \end{pmatrix} + \begin{pmatrix} c_2 \\ -ic_2 \end{pmatrix} = 0$$

$$\Rightarrow c_1 + c_2 = c_1 - c_2 = 0 \Rightarrow c_1 = c_2 = 0$$

So $\vec{y}^{\,0}$ can be expanded $\vec{y}^{\,0} = A\vec{a} + B\vec{b}$ & thus

$$\vec{y}^{\,n} = M^n\vec{y}^{\,0} = AM^n\vec{a} + BM^n\vec{b} = A\alpha^n\vec{a} + B\beta^n\vec{b}$$

(d) Let $T = Nh$

$$\vec{y}(T) = A\alpha^{T/h}\vec{a} + B\beta^{T/h}\vec{b}$$

$$= A(1+ih)^{T/h} + B(1-ih)^{T/h}\vec{b}$$

$$(1+ih)^{T/h} = (1+iT/N)^N \to e^{iT} \text{ as } N\to\infty$$

Similarly $(1-ih)^{T/h} \to e^{-iT}$ as $N\to\infty$

So $\vec{y}(T) \to Ae^{iT}\vec{a} + Be^{-iT}\vec{b}$

which is recognized as the exact solution, $\vec{y}_{ex}(t)$. To determine the rate of convergence, we investigate

$$|\vec{y}_{ex}(t) - \vec{y}(t)| = \left| A\left[e^{iT} - (1+iT/N)^N\right]\vec{a} + B\left[e^{-iT} - (1-iT/N)^N\right]\vec{b} \right|$$

$$\le |A|\left|e^{iT} - (1+iT/N)^N\right| + |B|\left|e^{-iT} - (1-iT/N)^N\right|$$

$$e^{iT} = 1 + iT + \frac{(iT)^2}{2} + \cdots$$

$$(1+iT/N)^N = 1 + N(iT/N) + \frac{N(N-1)}{2}\left(\frac{iT}{h}\right)^2 + \cdots + \frac{N(N-1)(N-2)}{6}\left(\frac{iT}{N}\right)^3$$