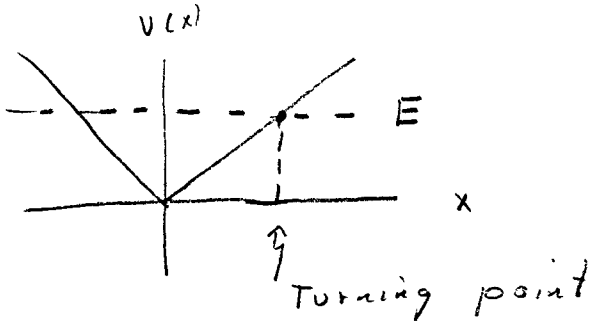
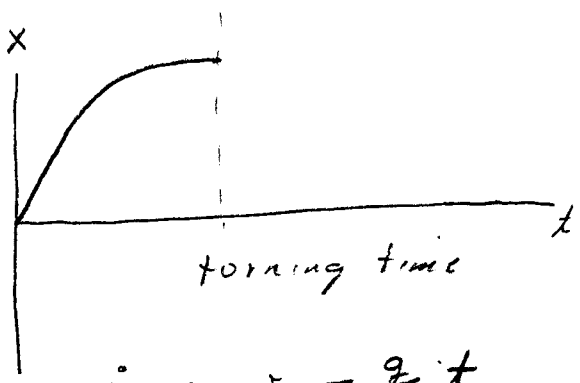
$$\Rightarrow \left(\frac{mv_0^2}{2} - gx\right)^{1/2} = -\left(\frac{2}{m}\right)^{1/2}\frac{g}{2}t + \left(\frac{1}{2}mv_0^2\right)^{1/2} \Rightarrow$$

$$\frac{1}{2}mv_0^2 - gx = \frac{2}{m}\frac{g^2}{4}t^2 - \left(\frac{2}{m}\right)^{1/2} gt\left(\frac{1}{2}mv_0^2\right)^{1/2} + \left(\frac{1}{2}mv_0^2\right)$$

or $$\boxed{x = +v_0 t - \frac{1}{2}\frac{g}{m}t^2}$$

This solution holds up to a turning time when $\dot{x} = 0$





$$\dot{x} = v_0 - \frac{g}{m}t$$

$$\dot{x} = 0 \Rightarrow \boxed{t = \frac{mv_0}{g}}$$

turning time

We get the solution beyond turning times by symmetry + periodic extension.