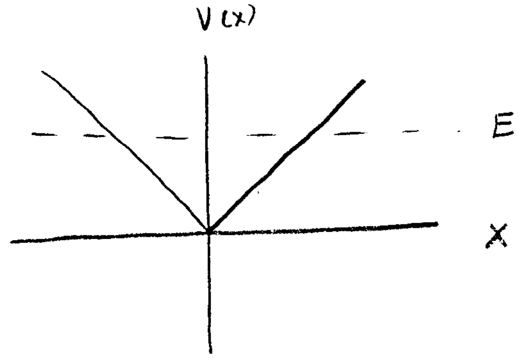
DCM 1.7 $V(x) = g|x|$



We know

$$\frac{1}{2} m \dot{x}^2 + V(x) = E \Rightarrow$$

$$\frac{1}{2} m \dot{x}^2 + g|x| = E.$$ Also, $x(0) = 0$ + $\dot{x}(0) = v_0 > 0$

$$\Rightarrow \boxed{E = \frac{1}{2} m v_0^2}$$. We also know from (3.27)

$$t - t^0 = \left(\frac{m}{2}\right)^{1/2} \int_{x^0}^{x} \frac{dx'}{\{E - V(x')\}^{1/2}}$$

Now since $x(0) = 0$, we get $x^0 = 0$ if we set $t^0 = 0$

$$\Rightarrow t = \left(\frac{m}{2}\right)^{1/2} \int_0^x \frac{dx'}{\{\frac{1}{2} m v_0^2 - g|x'|\}^{1/2}}.$$

We know $x > 0$ initially for t increasing since $v_0 > 0$.

Thus $|x| = x \Rightarrow$

$$t = \left(\frac{m}{2}\right)^{1/2} \int_0^x \frac{dx'}{\{\frac{1}{2} m v_0^2 - g x'\}^{1/2}} = \left(\frac{m}{2}\right)^{\frac{1}{2}} \left(-\frac{2}{g}\right) \left\{\frac{1}{2} m v_0^2 - g x'\right\}^{1/2} \Big|_0^x$$

Thus

$$t = -\left(\frac{m}{2}\right)^{1/2} \left(\frac{2}{g}\right) \left[\left(\frac{1}{2} m v_0^2 - g x\right)^{1/2} - \left(\frac{1}{2} m v_0^2\right)^{1/2}\right]$$