DCM 1.3 Need to show that

$$\vec r(t) = \vec r^{\circ} + (t-t^{\circ})\,\vec v^{\circ} + \frac{1}{m}\int_{t^{\circ}}^{t} dt'\,(t-t')\,\vec F(t') \qquad ★$$

satisfies $m\ddot{\vec r} = \vec F(t)$ with the initial conditions $\vec r(t^{\circ}) = \vec r^{\circ}$ and $\dot{\vec r}(t^{\circ}) = \vec v^{\circ}$.

Note that ★ involves only one integration, and not two, as expected!

i) Evidently
$$\vec r(t^{\circ}) = \vec r^{\circ} + \underbrace{(t^{\circ}-t^{\circ})}_{=0}\,\vec v + \underbrace{\frac{1}{m}\int_{t^{\circ}}^{t^{\circ}} dt'\,(t^{\circ}-t')\,\vec F(t')}_{=0}$$

$$\Rightarrow \vec r(t^{\circ}) = \vec r^{\circ}.$$

ii) Differentiating ★ ⇒

$$\dot{\vec r}(t) = \vec v^{\circ} + \underbrace{\frac{1}{m}(t-t)\,\vec F(t)}_{=0} + \frac{1}{m}\int_{t^{\circ}}^{t} dt'\,\vec F(t')$$

$$\Rightarrow \vec v(t) = \vec v^{\circ} + \frac{1}{m}\int_{t^{\circ}}^{t} dt'\,\vec F(t') \qquad ★☆$$

$$\vec v(t^{\circ}) = \vec v^{\circ}$$

iii) Differentiating ☆★ ⇒

$$\dot{\vec v}(t) = \frac{1}{m}\vec F(t) \Rightarrow m\ddot{\vec r} = \vec F$$