

Angular momentum conservation

gives

$$\boxed{r_1 m v_A = r_2 m v_B}$$

(2)

Solve equations (1) and (2) to find  $v_A + v_B$ :

$$\Rightarrow v_A^2 - \frac{2MG}{r_1} = v_B^2 - \frac{2MG}{r_2}$$

$$\Rightarrow r_1 v_A = r_2 v_B \Rightarrow v_B = \frac{r_1}{r_2} v_A$$

$$\Rightarrow v_A^2 - \frac{2MG}{r_1} = \left(\frac{r_1}{r_2}\right)^2 v_A^2 - \frac{2MG}{r_2}$$

$$\Rightarrow v_A^2 \left[1 - \left(\frac{r_1}{r_2}\right)^2\right] = 2MG \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow v_A^2 = 2MG \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{1 - \frac{r_1^2}{r_2^2}} = 2MG \frac{\left(\frac{r_2 - r_1}{r_1 r_2}\right)}{\frac{1}{r_2} (r_2^2 - r_1^2)}$$