

BDH
1.12

Consider circular orbit of radius r_1 . Then, from $m\vec{a} = \vec{F}$, one has (1/3)

$$m \frac{v_1^2}{r_1} = \frac{m M G}{r_1^2} \Rightarrow v_1^2 = \frac{M G}{r_1} \Rightarrow v_1 = \sqrt{\frac{M G}{r_1}} \quad (1)$$

Similarly, for circular orbit of radius r_2 ,

$$v_2 = \sqrt{\frac{M G}{r_2}} \quad (2)$$

Now consider the elliptic orbit. By the conservation of energy and angular momentum, these quantities have the same values at A and B. Consequently,

Energy conservation gives

$$\frac{1}{2} m v_A^2 - \frac{m M G}{r_1} = \frac{1}{2} m v_B^2 - \frac{m M G}{r_2} \quad (3)$$