b) Suppose $n = 2$: $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(1/2 + \frac{1}{2}) = \Gamma(1) = 1$

$$\therefore \boxed{\tau = \sqrt{\frac{2\pi m}{E}} \sqrt{\frac{E}{k}} \cdot \sqrt{\pi} = 2\pi \sqrt{\frac{m}{2k}}}$$

standard result for harmonic oscillator.

c) Suppose $V = k|x|^n$ & $n \to \infty$. Then $|x|^n \to 0$ for $|x| < 1$

$\to \infty$ for $|x| > 1$

$\therefore$ a square well of width 2.

$$\frac{1}{n} \frac{\Gamma(1/n)}{\Gamma(1/n + \frac{1}{2})} \longrightarrow \frac{1}{\Gamma(\frac{1}{2})} \frac{\Gamma(\frac{1}{n})}{n} \quad \text{as } n \to \infty.$$

But $\Gamma(x+1) = x\,\Gamma(x) \Rightarrow \Gamma(\frac{1}{n} + 1) = \frac{1}{n}\Gamma(\frac{1}{n}) \Rightarrow$

$$\lim_{n \to \infty} \frac{1}{n}\Gamma\left(\frac{1}{n}\right) = \Gamma(1) = 1 \quad \Rightarrow$$

$$\boxed{\tau_{n\to\infty} = 2\sqrt{\frac{2m}{E}}}$$

For a square well, $E = 1/2\, m v^2$, & period

$$\tau = \frac{\text{distance}}{\text{velocity}} = \frac{2 \times 2}{\sqrt{2E/m}} = 2\sqrt{\frac{2m}{E}}$$ which checks.

d) For $n=1$, $\Gamma(1) = 1$ & $\Gamma(1 + \frac{1}{2}) = \frac{1}{2}\Gamma(\frac{1}{2}) = \frac{1}{2}\sqrt{\pi} \Rightarrow$

$$\tau = 4\sqrt{\frac{2m}{E}}\,\frac{E}{k} = \frac{4\sqrt{2mE}}{k}.$$ But $E = \frac{1}{2} m v_0^2 \Rightarrow$

$2mE = m^2 v_0^2 \Rightarrow$

$$\boxed{\tau = \frac{4 m v_0}{k}}$$

which agrees with problem DCM1.7 when $k = g$