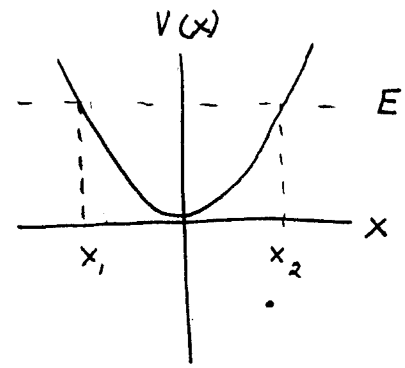
DCM 1.11 $\quad V(x) = k|x|^n$

a)



From (3.28) of chapter 1,

$$\tau = 2\sqrt{m/2}\int_{x_1}^{x_2}\frac{dx}{[E-V(x)]^{1/2}}$$

In this case $V(x) = V(-x)$ and $x_1 = -x_2$ $\Rightarrow$

$$\tau = 4\sqrt{m/2}\int_0^{x_2}\frac{dx}{[E-kx^n]^{1/2}}$$

Also, $V(x_2) = E \Rightarrow kx_2^n = E$.

Let $x = \lambda x_2$ + $E = kx_2^n$ $\Rightarrow$

$$\tau = 4\sqrt{m/2}\,\frac{x_2}{(kx_2^n)^{1/2}}\int_0^1\frac{d\lambda}{(1-\lambda^n)^{1/2}}$$

Use $x_2 = (E/k)^{1/n}$ and let $w = \lambda^n$, $\lambda = w^{1/n}$ $\Rightarrow$

$$\tau = 4\sqrt{\frac{m}{2}}\,\frac{1}{\sqrt{E}}\left(\frac{E}{k}\right)^{1/n}\frac{1}{n}\int_0^1 dw\, w^{1/n-1}\,(1-w)^{-1/2}.$$

But $\displaystyle\int_0^1 x^{a-1}(1-x)^{b-1}\,dx = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$ $\Rightarrow$

$$\tau = 4\sqrt{\frac{m}{2E}}\left(\frac{E}{k}\right)^{1/n}\frac{1}{n}\,\frac{\Gamma(1/n)\,\Gamma(1/2)}{\Gamma(1/n+\frac{1}{2})}.$$

Also, $\Gamma(1/2) = \sqrt{\pi}$ $\Rightarrow$

$$\boxed{\tau = \frac{2}{n}\sqrt{\frac{2\pi m}{E}}\left(\frac{E}{k}\right)^{1/n}\frac{\Gamma(1/n)}{\Gamma(1/n+\frac{1}{2})}}$$