Dragt 99 cont. Next need to sum the series.

Use the trig identity

$$\cos\frac{n\pi v}{L}t \,\sin\frac{n\pi x}{L} = \frac{1}{2}\left[\sin\left(\frac{n\pi x}{L} + \frac{n\pi v}{L}t\right) + \sin\left(\frac{n\pi x}{L} - \frac{n\pi v}{L}t\right)\right]$$

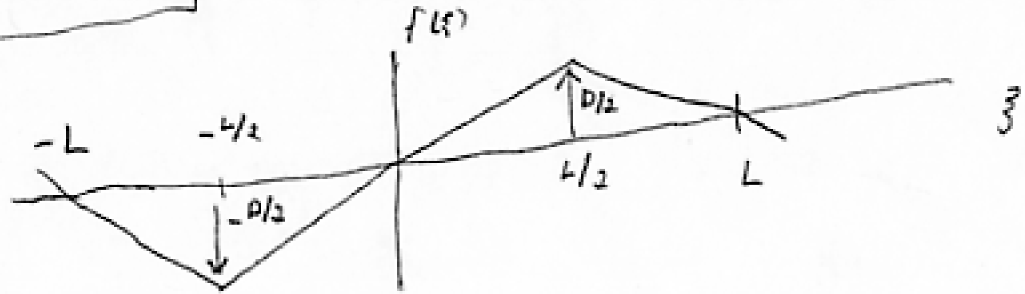
$$= \frac{1}{2}\left\{\sin\left[\left(\frac{n\pi}{L}\right)(x+vt)\right] + \sin\left[\left(\frac{n\pi}{L}\right)(x-vt)\right]\right\}.$$ This gives

$$\boxed{g(x,t) = f(x+vt) + f(x-vt) \quad \text{where} \quad f(\xi) = \sum_1^\infty \frac{8D}{\pi^2 n^2}\sin\frac{n\pi}{2}\left(\frac{1}{2}\right)\sin\left[\left(\frac{n\pi}{L}\right)\xi\right].}$$

Next we observe that $g(x,0) = \sum_1^\infty \frac{8D}{\pi^2 n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{L}$ and therefore

$$\boxed{f(\xi) = \frac{1}{2}g(\xi,0).}$$

But, $g(\xi,0)$ is known, and therefore $f(\xi)$ has the graph




Note that

a) $f(\xi)$ is odd because it is given by a sine series.

b) $f(\xi)$ has period $2L$.