Dragt 9.9

We have generally $q(x,t) = \sum_{1}^{\infty} \left[a_n(0) \cos \omega_n t + \dot{a}_n(0)\, \omega_n^{-1} \sin \omega_n t \right] \sin \frac{n\pi x}{L}$

with $\omega_n = n\pi v L^{-1}$ and

$$a_n(0) = \frac{2}{L} \int_0^L q(x,0) \sin \frac{n\pi x}{L}\, dx, \quad \dot{a}_n(0) = \frac{2}{L} \int_0^L \dot{q}(x,0) \sin \frac{n\pi x}{L}\, dx .$$

In our case, $\boxed{\dot{a}_n(0) = 0}$ since $\left.\frac{\partial q}{\partial t}\right|_{t=0} = 0$, and

$$a_n(0) = \frac{2}{L} \left\{ \int_0^{L/2} \frac{2Dx}{L} \sin \frac{n\pi x}{L}\, dx + \int_{L/2}^{L} 2D \left[1 - \frac{x}{L} \right] \sin \frac{n\pi x}{L}\, dx \right\}$$

Doing the integrals $\Rightarrow$ $\boxed{a_n(0) = \frac{8D}{\pi^2 n^2} \sin \frac{n\pi}{2}}$ $\Rightarrow$

$$\boxed{q(x,t) = \sum_{1}^{\infty} \frac{8D}{\pi^2 n^2} \sin \frac{n\pi}{2} \cos \frac{n\pi v}{L} t \sin \frac{n\pi x}{L}}$$

. Note that only the odd harmonics are excited, and they have relative amplitudes $\sim n^{-2}$.