Dragt 9.8

We have $\frac{\partial^2 q}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 q}{\partial t^2}$ with $v^2 = \frac{\tau}{\rho}$. Write

$q(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, e^{ikx} f(k,t)$. Then the wave equation $\Rightarrow$

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} + k^2 f = 0 \Rightarrow f(k,t) = a(k)\cos\omega t + b(k)\frac{1}{\omega}\sin\omega t$$

with $\boxed{\omega = kv}$ $\Rightarrow q(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, e^{ikx}\left[a(k)\cos\omega t + \frac{b(k)}{\omega}\sin\omega t\right]$

and $\frac{\partial q}{\partial t} = \frac{1}{\sqrt{2\pi}}\int dk\, e^{ikx}\left[-\omega\, a(k)\sin\omega t + b(k)\cos\omega t\right]$. But,

$$q(x,0) = A e^{-x^2/B} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, e^{ikx} a(k) \Rightarrow \boxed{a(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, e^{-ikx} A e^{-\frac{x^2}{B}}}$$

and $0 = \left.\frac{\partial q}{\partial t}\right|_{t=0} = \frac{1}{\sqrt{2\pi}}\int dk\, e^{ikx} b(k) \Rightarrow \boxed{b(k) = 0}$

$$\therefore \boxed{q(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, e^{ikx}\cos kvt\, \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx' e^{-ikx'} A e^{-\frac{x'^2}{B}}}$$