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We still need to find the normalization constants α^j.

We have $(n^j, M n^j) = 1 = m(n^j, n^j) = m(\alpha^j)^2 \sum_{k=1}^{N} \left(\sin \frac{k\pi j}{N+1}\right)^2$

$$\therefore\ m^{-1}(\alpha^j)^{-2} = \sum_{k=0}^{N} \sin^2 \frac{k\pi j}{N+1} = \frac{1}{2} \sum_{k=0}^{N} 1 - \cos \frac{2k\pi j}{N+1}$$

$$= \frac{N+1}{2}. \left\{ \text{Note that } \sum_{k=0}^{N} \cos \frac{2\pi k j}{N+1} = \operatorname{Re} \sum_{k=0}^{N} e^{\frac{2\pi k j i}{N+1}} \right.$$

$$\left. = \operatorname{Re}\left[\frac{e^{\frac{2\pi (N+1) j i}{N+1}} - 1}{e^{\frac{2\pi j i}{N+1}} - 1} \right] = 0. \right\} \quad \therefore\ \forall\ \alpha^j \text{ are equal,}$$

and we can write $\boxed{q_j = \frac{1}{\sqrt{m}} \left(\frac{2}{N+1}\right)^{\frac{1}{2}} \sum_{k=1}^{N} Q_k \sin \frac{k\pi j}{N+1}}$

as the normal mode decomposition.