Let us look for solutions to the general difference equation $-n_{k-1}^j + 2n_k^j - n_{k+1}^j = \gamma_j n_k$. Make the ansatz

$$n_k^j \propto e^{k\beta^j} \Rightarrow -e^{(k-1)\beta^j} + 2e^{k\beta^j} - e^{(k+1)\beta^j} = \gamma_j e^{k\beta^j}$$

or $-e^{-\beta^j} + 2 - e^{\beta^j} = \gamma_j$ or $\boxed{2-\gamma_j = 2\cosh \beta^j}$

Likewise, $e^{-k\beta^j}$ is also a solution with the same result

Thus, general solution is $n_k^j = A^j e^{k\beta^j} + B^j e^{-k\beta^j}$.

Next impose the boundary conditions $n_0^j = 0$ and $n_{N+1}^j = 0$ to get the right conditions at the ends of the "string".

Thus, $A^j = -B^j$, and $e^{(N+1)\beta^j} - e^{-(N+1)\beta^j} = 0$

or $e^{2(N+1)\beta^j} = 1 \Rightarrow \boxed{(N+1)\beta^j = n\pi i}$ $\quad n = 1, \cdots N$