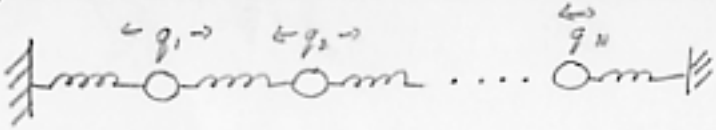
Draft 97



$$T = \sum_1^N \tfrac{1}{2} m \dot{q}_i^2$$

$$V = \tfrac{1}{2} k \left[q_1^2 + (q_1 - q_2)^2 + \cdots + (q_{N-1} - q_N)^2 + q_N^2 \right]$$

$$\therefore \quad M = m \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}, \quad A = k \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \cdots \\ -1 & 2 & -1 & 0 & 0 & \cdots \\ 0 & -1 & 2 & -1 & 0 & \cdots \\ 0 & 0 & -1 & 2 & -1 & 0 \cdots \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & & -1 \\ & & & & -1 & 2 \end{pmatrix} = k\Gamma$$

We need to find vectors n^j and "eigenvalues" λ_j such that

$$A n^j = \lambda_j M n^j, \quad \text{or}$$

$$k \Gamma n^j = \lambda_j m n^j \quad \text{or} \quad \Gamma n^j = \gamma_j n^j \quad \text{where} \quad \boxed{\gamma_j = \frac{m}{k} \lambda_j}$$

Writing the equations out in components ⇒

$$2 n_1^j - n_2^j = \gamma_j n_1$$

$$-n_1^j + 2 n_2^j - n_3^j = \gamma_j n_2$$

$$-n_{k-1}^j + 2 n_k^j - n_{k+1}^j = \gamma_j n_k$$

$$-n_{N-2}^j + 2 n_{N-1}^j - n_N^j = \gamma_j n_{N-1}$$

$$-n_{N-1}^j + 2 n_N^j = \gamma_j n_N^j$$