



The wave equation is satisfied by functions $f(x-vt)$ and $g(x+vt)$. Write

$$g(x,t) = [f(x-vt) + g(x+vt)] \leftarrow \begin{matrix} \text{most general} \\ \text{solution} \end{matrix}$$

Then the boundary condition $g=0$ at $x=0 \Rightarrow$
 $0 = g(0,t) = f(-vt) + g(vt) \Rightarrow g(vt) = -f(-vt)$

Thus
$$g(x,t) = f(x-vt) - f(-x-vt) \quad (a)$$

We next impose the other boundary condition

$$g(L,t) = 0 = f(L-vt) - f(-L-vt)$$

$$\Rightarrow f(L-vt) = f(-L-vt) \Rightarrow f(2L-vt) = f(-vt)$$

upon setting $t = t' - L/v$. Thus, f

is periodic with period $2L$. We next

define f on the interval $[0, 2L]$ by the

rule
$$f(x) = g(x,0) \quad \text{for } 0 \leq x \leq L \quad (b)$$

$$f(x) = 0 \quad \text{for } L \leq x \leq 2L.$$

Then looking at (a) shows that we get a