Draft 96

We begin with a discussion of the wave equation.

The wave equation reads $\frac{\partial^2 q}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 q}{\partial t^2} = 0$

Let $\xi = x + vt$ + $\eta = x - vt$ be new variables $\Rightarrow$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial t} = v\left[\frac{\partial}{\partial \xi} - \frac{\partial}{\partial \eta}\right]$$

$$\therefore \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2} + 2\frac{\partial}{\partial \xi}\frac{\partial}{\partial \eta} + \frac{\partial^2}{\partial \eta^2}, \quad \frac{1}{v^2}\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial \xi^2} - 2\frac{\partial}{\partial \xi}\frac{\partial}{\partial \eta} + \frac{\partial^2}{\partial \eta^2}$$

and $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) q = 4\frac{\partial^2}{\partial \xi \partial \eta} q = 0.$

Integrating $\frac{\partial^2}{\partial \xi \partial \eta} q = 0 \Rightarrow \frac{\partial}{\partial \xi}\frac{\partial q}{\partial \eta} = 0 \Rightarrow \frac{\partial q}{\partial \eta} = h(\eta)$

$\Rightarrow q = f(\eta) + g(\xi) = f(x - vt) + g(x + vt).$