Rewriting, we get

$$\mathcal{L} = \frac{1}{2} m \dot{\vec{R}}_{CM}^{\,2} - \frac{3}{2} k \vec{R}_{CM}^{\,2} + \frac{ma^2}{48}\left(\dot{\epsilon}_1^2 + \dot{\epsilon}_2^2\right) - \frac{kad}{4}\left(\epsilon_1^2 + \epsilon_2^2\right) + \frac{ma^2}{24}\dot{\epsilon}_3^2 - \frac{kad}{2}\epsilon_3^2 .$$

We see that the translational degrees of freedom are all degenerate, and have a frequency $\boxed{\omega_{trans} = \sqrt{\frac{3k}{m}}}$. The rotational modes are also all degenerate, and $\boxed{\omega_{rot} = \sqrt{\left(\frac{k}{m}\frac{d}{a}\right)12}}$