

$$= S_y - (\hat{e}_x \cdot \hat{f}_b) (\hat{e}_j \cdot \hat{f}_b) . \quad \text{Thus, } T_y = 3 S_y - \sum_b (\hat{e}_x \cdot \hat{f}_b) (\hat{e}_j \cdot \hat{f}_b) .$$

We see immediately  $T_{23} = T_{32} = 0$  for  $k \neq 3$ , and  $T_{33} = 3$ ,

since  $\hat{e}_3 \cdot \hat{f}_b = 0 \forall b$ . Next,

$$T_{12} = T_{21} = - \sum_b (\hat{e}_1 \cdot \hat{f}_b) (\hat{e}_2 \cdot \hat{f}_b) = - \left\{ 1 \times 0 + (-\frac{1}{2}) (\frac{\sqrt{3}}{2}) + (-\frac{1}{2}) (-\frac{\sqrt{3}}{2}) \right\} = 0 .$$

Finally,  $T_{11} = 3 - \sum_b (\hat{e}_1 \cdot \hat{f}_b) (\hat{e}_1 \cdot \hat{f}_b) = 3 - \left\{ 1 + \frac{1}{4} + \frac{1}{4} \right\} = \frac{3}{2}$

$$T_{22} = 3 - \sum_b (\hat{e}_2 \cdot \hat{f}_b) (\hat{e}_2 \cdot \hat{f}_b) = 3 - \left\{ 0 + \frac{3}{4} + \frac{3}{4} \right\} = \frac{3}{2} .$$

Thus  $V_{Rot} = \frac{k a d}{4} \left\{ 2 \epsilon_3^2 + \epsilon_1^2 + \epsilon_2^2 \right\} + const + O(\epsilon^3)$

and

$$\mathcal{L} = \left[ \frac{1}{2} m \dot{\vec{R}}_{CM}^2 - \frac{3}{2} k \vec{R}_{CM}^2 \right] + \left[ \frac{m a^2}{48} (\dot{\epsilon}_1^2 + \dot{\epsilon}_2^2 + 2 \dot{\epsilon}_3^2) - \frac{k a d}{4} (\epsilon_1^2 + \epsilon_2^2 + 2 \epsilon_3^2) \right] + O(\epsilon^3) .$$