We must still evaluate the rotational part of the potential energy: $V_{Rot} = -\frac{kad}{3} \sum_b \hat{f}_b \cdot (\mathcal{R} \hat{f}_b)$. Using the parameterization for $\mathcal{R}$, we get

$$V_{Rot} = -\frac{kad}{3} \sum_b \hat{f}_b \cdot \hat{f}_b - \frac{kad}{6} \sum_{i,j} \epsilon_i \epsilon_j \sum_b \hat{f}_b \cdot (J_i J_j \hat{f}_b) + O(\epsilon^3)$$

Here we used $\hat{f}_b \cdot (J_i \hat{f}_b) = 0$ since $\tilde{J}_i = -J_i$. Thus,

$$V_{Rot} = \frac{kad}{6} \sum_{i,j} \epsilon_i \epsilon_j \sum_b (\hat{e}_i \times \hat{f}_b) \cdot (\hat{e}_j \times \hat{f}_b) + \text{constant} + O(\epsilon^3)$$

where we used $J_j \hat{f}_b = (\hat{e}_j \cdot \vec{J}) \hat{f}_b = \hat{e}_j \times \hat{f}_b$. Let us evaluate the tensor T_{ij} given by $T_{ij} = \sum_b (\hat{e}_i \times \hat{f}_b) \cdot (\hat{e}_j \times \hat{f}_b)$.

We have $(\hat{e}_i \times \hat{f}_b) \cdot (\hat{e}_j \times \hat{f}_b) = [(\hat{e}_i \times \hat{f}_b) \times \hat{e}_j] \cdot \hat{f}_b$

Using $A \cdot (B \times C) = (A \times B) \cdot C$

$= -[\hat{e}_j \times (\hat{e}_i \times \hat{f}_b)] \cdot \hat{f}_b = -[\hat{e}_i (\hat{e}_j \cdot \hat{f}_b) - \delta_{ij} \hat{f}_b] \cdot \hat{f}_b$