Dragt 94 We have $I_{\alpha\gamma} = \int d^3\vec{r}\, \rho(\vec{r}) \left\{ \delta_{\alpha\gamma} \vec{r}^{\,2} - r_\alpha r_\gamma \right\}$. Looking at the figure given with the problem, we define a function $\Delta(\vec{r})$ with the properties: (a) $\Delta(\vec{r}) = 1$ when $\vec{r}$ contained in triangle

(b) $\Delta(\vec{r}) = 0$ when $\vec{r}$ not " " "

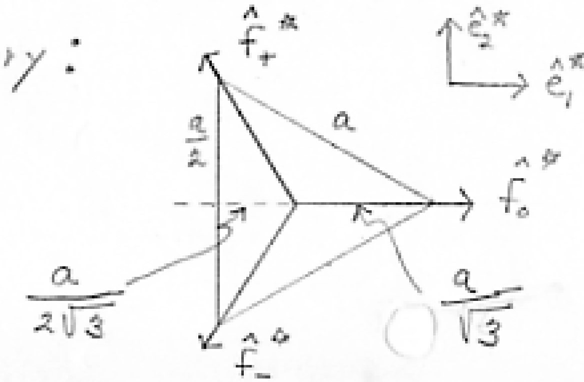
Then, $\rho(\vec{r}) = mA^{-1}\delta(z)\,\Delta(\vec{r})$ where A is the Area of the triangle.

Proof: $\int \rho(\vec{r})\, d^3\vec{r} = mA^{-1}\int dz\, \delta(z) \int dx\,dy\, \Delta(\vec{r}) = mA^{-1}\int_{\text{triangle}} dx\,dy = mA^{-1}A = m.$

It is now evident that $I_{\alpha,3} = I_{3,\alpha} = 0$ for $\alpha \neq 3$ since

$I_{\alpha,3} = -mA^{-1}\int dx\,dy\, \Delta(\vec{r}) \int dz\, \delta(z)\, z\, r_\alpha = 0$. Also, $I_{12} = -mA^{-1}\int_{\text{triangle}} dx\,dy\; xy = 0$ by symmetry. <u>Thus</u>, <u>the axes</u> $\hat{e}_\gamma^{\,*}$ <u>are principal axes.</u> Next observe that any one of the three axes $\hat{f}_+^{\,*}$, $\hat{f}_-^{\,*}$, and $\hat{f}_0^{\,*}$ shown below are <u>principal axes</u> having the <u>same moment</u> of inertia by symmetry:

$$\boxed{\text{Note} \quad \sum_{b=0,+,-} \hat{f}_b^{\,*} = 0}$$



$$\boxed{\begin{aligned} \hat{f}_0^{\,*} &= \hat{e}_1^{\,*} \\ \hat{f}_\pm^{\,*} &= -\tfrac{1}{2}\hat{e}_1^{\,*} \pm \tfrac{\sqrt{3}}{2}\hat{e}_2^{\,*} \end{aligned}}$$