$\theta_1, \theta_2, \theta_3$ are the normal coordinates
Inverting gives

$$Q_1 = (\phi_1 - \phi_3)\frac{\sqrt{MR^2}}{2}$$

$$Q_2 = (\phi_1 - \sqrt{2}\phi_2 + \phi_3)\frac{\sqrt{MR^2}}{2\sqrt{2}} \qquad (B)$$

$$Q_3 = (\phi_1 + \sqrt{2}\phi_2 + \phi_3)\frac{\sqrt{MR^2}}{2\sqrt{2}}$$

(b) The Q_i are given by $Q_i(t) = Q_i(0)\cos\omega_i t + \frac{\dot{Q}_i(0)}{\omega_i}\sin\omega_i t$

For the initial conditions given

$$\phi_i(t=0) = 0 \xRightarrow{\text{looking at (B)}} Q_i(0) = 0$$

$$\dot{\phi}_1(0) = \frac{J}{\frac{1}{2}MR^2} \qquad \dot{\phi}_2(0) = \dot{\phi}_3(0) = 0$$

$$\therefore \quad \dot{Q}_1(0) = \frac{2J}{MR^2}\frac{\sqrt{MR^2}}{2} = \frac{J}{\sqrt{MR^2}}$$

$$\dot{Q}_2(0) = \frac{2J}{MR^2}\frac{\sqrt{MR^2}}{2\sqrt{2}} = \frac{J}{\sqrt{2MR^2}}$$

$$\dot{Q}_3(0) = \frac{2J}{MR^2}\frac{\sqrt{MR^2}}{2\sqrt{2}} = \frac{J}{\sqrt{2MR^2}}$$

Putting this in (A) $\Longrightarrow$

$$\phi_2 = \frac{1}{\sqrt{MR^2}}\left(-\frac{J}{\sqrt{2MR^2}}\frac{1}{\omega_2}\sin\omega_2 t + \frac{J}{\sqrt{2MR^2}}\frac{1}{\omega_3}\sin\omega_3 t\right)$$

$$\phi_2 = \frac{J}{MR^2\sqrt{2}}\left(\frac{1}{\omega_3}\sin\omega_3 t - \frac{1}{\omega_2}\sin\omega_2 t\right)$$