$$u_3 = \begin{pmatrix} a_3 \\ \sqrt{2}\,b_3 \\ a_3 \end{pmatrix}$$

Normalization $\Rightarrow \quad 4a_3^2 I = 1 \qquad a_3 = \frac{1}{2\sqrt{I}} = \frac{1}{\sqrt{2MR^2}}$

$\therefore \quad u_3 = \frac{1}{\sqrt{2MR^2}}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$ Note that u_1, u_2 and u_3 are orthogonal as required.

Let $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Then $q = SQ$.

$$\phi_1 = S_{11}Q + S_{12}Q_2 + S_{13}Q_3 \quad \text{etc.}$$

where $S_{11} = (e_1, u_1)$, $S_{ij} = (e_i, u_j)$

$$u_1 = \frac{1}{\sqrt{MR^2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad u_2 = \frac{1}{\sqrt{2MR^2}}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} \qquad u_3 = \frac{1}{\sqrt{2MR^2}}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

$$S_{11} = \frac{1}{\sqrt{MR^2}} \qquad S_{12} = \frac{1}{\sqrt{2MR^2}} \qquad S_{13} = \frac{1}{\sqrt{2MR^2}}$$

$$S_{21} = 0 \qquad S_{22} = -\frac{1}{\sqrt{MR^2}} \qquad S_{23} = \frac{1}{\sqrt{MR^2}}$$

$$S_{31} = -\frac{1}{\sqrt{MR^2}} \qquad S_{32} = \frac{1}{\sqrt{2MR^2}} \qquad S_{33} = \frac{1}{\sqrt{2MR^2}}$$

$$\phi_1 = \frac{1}{\sqrt{MR^2}}\left(\theta_1 + \frac{\theta_2}{\sqrt{2}} + \frac{\theta_3}{\sqrt{2}}\right)$$

$$\phi_2 = \frac{1}{\sqrt{MR^2}}\left(-\theta_2 + \theta_3\right) \qquad \text{(A)}$$

$$\phi_3 = \frac{1}{\sqrt{MR^2}}\left(-\theta_1 + \frac{\theta_2}{\sqrt{2}} + \frac{\theta_3}{\sqrt{2}}\right)$$