

$$\therefore U_1 = \frac{1}{\sqrt{MR^2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Similarly, using  $AU_2 = \lambda_2 M U_2$

$$2a_2 - b_2 = (2 + \sqrt{2})a_2 \Rightarrow b_2 = -\sqrt{2}a_2$$

$$-a_2 + 2b_2 - c_2 = (2 + \sqrt{2})b_2 \Rightarrow c_2 = a_2$$

$$-b_2 + 2c_2 = (2 + \sqrt{2})c_2 \Rightarrow c_2 = a_2$$

$$U_2 = \begin{pmatrix} a_2 \\ -\sqrt{2}a_2 \\ a_2 \end{pmatrix}$$

Normalization  $(U_2, MU_2) = 1$  gives

$$I(a_2, -\sqrt{2}a_2, a_2) \begin{pmatrix} a_2 \\ -\sqrt{2}a_2 \\ a_2 \end{pmatrix} = 1$$

$$\text{i.e. } a_2^2 I + 2a_2^2 I + a_2^2 I = 4a_2^2 I = 1$$

$$\therefore a_2 = \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2MR^2}}$$

$$U_2 = \frac{1}{\sqrt{2MR^2}} \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$

Similarly for  $U_3$ , we have

$$2a_3 - b_3 = (2 - \sqrt{2})a_3 \Rightarrow b_3 = \sqrt{2}a_3$$

$$-a_3 + 2b_3 - c_3 = (2 - \sqrt{2})b_3 \Rightarrow a_3 = c_3$$

$$-b_3 + 2c_3 = (2 - \sqrt{2})c_3 \Rightarrow a_3 = c_3$$