$\therefore \quad \lambda_1 = \frac{2k}{\ell I} \qquad \lambda_2 = (2+\sqrt{2})\frac{k}{\ell I} \qquad \lambda_3 = (2-\sqrt{2})\frac{k}{\ell I}$

The normal frequencies are

$$\omega_1^2 = \lambda_1 \quad \omega_2^2 = \lambda_2 \quad \omega_3^2 = \lambda_3$$

Next we try to find a set of eigenvectors satisfying $\quad A u_i = \lambda_i M u_i$

$$\frac{k}{\ell}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = \lambda_1 \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$$

Equating:

$$2\frac{k}{\ell} a_1 - \frac{k}{\ell} b_1 = \frac{2k}{\ell I} \cdot I a_1$$

$$\Rightarrow \quad 2a_1 - b_1 = 2a_1 \qquad \Rightarrow b_1 = 0$$

Similarly $\quad -a_1 + 2b_1 - c_1 = 2b_1 \Rightarrow a_1 = -c_1$

$$-b_1 + 2c_1 = 2c_1 \qquad \Rightarrow b_1 = 0$$

$$\therefore \quad u_1 = \begin{pmatrix} a_1 \\ 0 \\ -a_1 \end{pmatrix}$$

Imposing $\quad (u_1, M u_1) = 1$

$$a_1^2 I + a_1^2 I = 1$$

$$a_1 = \frac{1}{\sqrt{2I}} = \frac{1}{\sqrt{2 \cdot \frac{MR^2}{2}}} = \frac{1}{\sqrt{MR^2}}$$