Then $T = \frac{1}{2}(\dot{\phi}, M\dot{\phi})$ and $V = \frac{1}{2}(\phi, A\phi)$

with $M = \frac{1}{2}MR^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$A = \frac{k}{L}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

We next find the polynomial given by setting

$$|A - \lambda M| = 0$$

$$\Rightarrow \begin{vmatrix} \frac{2k}{L} - \lambda I & -\frac{k}{L} & 0 \\ -\frac{k}{L} & \frac{2k}{L} - \lambda I & \frac{-k}{L} \\ 0 & \frac{-k}{L} & \frac{2k}{L} - \lambda I \end{vmatrix} = 0$$

where $I = \frac{1}{2}MR^2$

or $\left(\frac{2k}{L} - \lambda I\right)\left\{\left(\frac{2k}{L} - \lambda I\right)^2 - 2\frac{k^2}{L^2}\right\} = 0$

or $\left(\frac{2k}{L} - \lambda I\right)\left\{\left(\frac{2k}{L}\right)^2 + \lambda^2 I^2 - 2\frac{k^2}{L^2} - \frac{4k\lambda I}{L}\right\} = 0$

$\Rightarrow \boxed{\lambda = \frac{2k}{LI}}$ or $\lambda = \dfrac{\frac{4kI}{L} \pm \sqrt{\left(\frac{4kI}{L}\right)^2 - 4I^2 \cdot \frac{2k^2}{L^2}}}{2I^2}$

$= \frac{2k}{LI} \pm \frac{2\sqrt{2}}{L} kI \cdot \frac{1}{2I^2}$