



If a rod is twisted by an angle ϕ , then the twist per unit length is $\frac{\phi}{L}$

$$\therefore \text{Torque} = \frac{k\phi}{L} \quad \text{and} \quad V = \frac{1}{2} \frac{k}{L} \phi^2$$

The moment of inertia of a disc along the rod through its center is $I = \frac{1}{2} MR^2$

R : radius of the disc.

Since the rods are rigidly fastened the kinetic energy

$$T = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} I_3 \omega_3^2$$

$$= \frac{1}{2} \cdot \frac{1}{2} MR^2 (\dot{\phi}_1^2 + \dot{\phi}_2^2 + \dot{\phi}_3^2)$$

The potential energy.

$$V = \frac{k}{2L} [\phi_1^2 + (\phi_1 - \phi_2)^2 + (\phi_2 - \phi_3)^2 + \phi_3^2]$$

$$= \frac{k}{2L} [2\phi_1^2 + 2\phi_2^2 + 2\phi_3^2 - 2\phi_1\phi_2 - 2\phi_2\phi_3]$$

let $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$