

$$\theta_1 = \frac{\theta_1^0}{2} [\cos \omega_+ t + \cos \omega_- t]$$

$$\theta_2 = \frac{\theta_1^0}{2} \sqrt{\frac{m_1 + m_2}{m_2}} [\cos \omega_+ t - \cos \omega_- t]$$

or, using trig
formulas,

$$\theta_1 = \theta_1^0 \cos\left[\frac{1}{2}(\omega_+ + \omega_-)t\right] \cos\left[\frac{1}{2}(\omega_+ - \omega_-)t\right]$$

$$\theta_2 = \theta_1^0 \sin\left[\frac{1}{2}(\omega_+ + \omega_-)t\right] \sin\left[\frac{1}{2}(\omega_+ - \omega_-)t\right] \otimes \sqrt{\frac{m_1 + m_2}{m_2}}$$

This form
shows the
beat phenomena.

An amazingly simple answer considering the earlier complexities!