$$(n^+, M n^-) \propto \left(1, \sqrt{\frac{m_1+m_2}{m_2}}\right) \begin{pmatrix} (m_1+m_2)\,l^2 - m_2 l^2 \sqrt{\frac{m_1+m_2}{m_2}} \\ m_2 l^2 - m_2 l^2 \sqrt{\frac{m_1+m_2}{m_2}} \end{pmatrix} = l^2 \left\{ (m_1+m_2) - \sqrt{(m_1+m_2)m_2} + \sqrt{m_2(m_1+m_2)} - (m_1+m_2) \right\}$$

Finally, we check

$= 0$ as expected

Now to find solution fitting initial conditions:

We have $\Theta_i = \sum_j N_{ij} Q_j$, $N_{ij} = (e^i, n^j)$ & $Q_j = C_j \cos(\omega_j t + \delta_j)$.

For the initial conditions of the problem, $\Theta_1 = \Theta_1^\circ$, $\Theta_2 = 0$

$\dot\Theta_1 = 0$, $\dot\Theta_2 = 0$

Thus,

$$\Theta_1^\circ = (e^1, n^+) C_+ \cos(\omega_+ \overset{0}{t} + \delta_+) + (e^1, n^-) C_- \cos(\omega_- \overset{0}{t} + \delta_-)$$

$$0 = (e^2, n^+) C_+ \cos(\omega_+ 0 + \delta_+) + (e^2, n^-) C_- \cos(\omega_- 0 + \delta_-)$$

$$0 = -(e^1, n^+) \omega_+ C_+ \sin(\omega_+ 0 + \delta_+) - (e^1, n^-) \omega_- C_- \cos(\omega_- 0 + \delta_-)$$

$$0 = -(e^2, n^+) \quad \cdot\cdot \quad - (e^2, n^-) \quad \cdot\cdot$$