

We make a preliminary normalization by setting $n_1^\pm = 1$. Page 4/9

Then, $n_2^\pm [1 \pm \sqrt{\frac{m_2}{m_1+m_2}}] = 1 + n_2^\pm \Rightarrow \pm n_2^\pm \sqrt{\frac{m_2}{m_1+m_2}} = 1$

$\Rightarrow n_2^\pm = \pm \sqrt{\frac{m_1+m_2}{m_2}}$, $n_1^\pm = 1$ is unnormalized.

Next require $(n^\alpha, H n^\beta) = \delta_{\alpha\beta}$

$(n^T, H n^T) = (1, \sqrt{\frac{m_1+m_2}{m_2}}) \begin{pmatrix} (m_1+m_2)L^2 & m_2 L^2 \\ m_2 L^2 & m_2 L^2 \end{pmatrix} \begin{pmatrix} 1 \\ \sqrt{\frac{m_1+m_2}{m_2}} \end{pmatrix}$

$= (1, \sqrt{\frac{m_1+m_2}{m_2}}) \begin{pmatrix} (m_1+m_2)L^2 + m_2 L^2 \sqrt{\frac{m_1+m_2}{m_2}} \\ m_2 L^2 + m_2 L^2 \sqrt{\frac{m_1+m_2}{m_2}} \end{pmatrix} = L^2 \left[\begin{matrix} (m_1+m_2) + \sqrt{(m_1+m_2)m_2} \\ \sqrt{m_2(m_1+m_2)} + m_1+m_2 \end{matrix} \right]$

$= 2L^2 \left[(m_1+m_2) + \sqrt{(m_1+m_2)m_2} \right]$