Goldstein 6.4 cont.

or

$$g = \lambda l \left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right] \Rightarrow \boxed{\lambda_\pm = \frac{g}{l}\left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right]^{-1}}$$

Finally,

$$\boxed{\omega_\pm = \lambda_\pm^{\frac{1}{2}} = \sqrt{\frac{g}{l}}\left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right]^{-\frac{1}{2}}}$$

Note that as $m_2 \to 0$, $\omega_\pm \longrightarrow \sqrt{g/l}$

To find the normal modes, we look for vectors which satisfy

$$A \eta^\pm = \lambda_\pm M \eta^\pm \Rightarrow \boxed{\begin{pmatrix} (m_1+m_2) g l & 0 \\ 0 & m_2 g l \end{pmatrix}\begin{pmatrix} \eta_1^\pm \\ \eta_2^\pm \end{pmatrix} = \frac{g}{l}\left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right]^{-1} \begin{pmatrix} (m_1+m_2) l^2 & m_2 l^2 \\ m_2 l^2 & m_2 l^2 \end{pmatrix}\begin{pmatrix} \eta_1^\pm \\ \eta_2^\pm \end{pmatrix}}$$

$$\Rightarrow \eta_1^\pm (m_1+m_2) g l = g/l \left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right]^{-1}\left[(m_1+m_2) l^2 \eta_1^\pm + m_2 l^2 \eta_2^\pm\right]$$

$$\eta_2^\pm \, m_2 g l = g/l \left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right]^{-1}\left[m_2 l^2 \eta_1^\pm + m_2 l^2 \eta_2^\pm\right]$$

The 2^{nd} equation $\Rightarrow$ $\eta_2^\pm = \left[1 \pm \sqrt{\frac{m_2}{m_1+m_2}}\right]^{-1}\left[\eta_1^\pm + \eta_2^\pm\right]$