Goldstein 6.4 cont.

Comparing with the standard Lagrangian of the Handout Notes,

$$\mathcal{L} = \frac{1}{2}\sum_{ij} m_{ij}\,\dot{q}_i\dot{q}_j - \frac{1}{2}\sum_{ij} a_{ij}\,q_i q_j \text{, we get}$$

$$M = \begin{pmatrix} (m_1+m_2)\ell^2 & m_2\ell^2 \\ m_2\ell^2 & m_2\ell^2 \end{pmatrix}, \quad A = \begin{pmatrix} (m_1+m_2)g\ell & 0 \\ 0 & m_2 g\ell \end{pmatrix}$$

$$\det(A-\lambda M) = \begin{vmatrix} (m_1+m_2)g\ell - \lambda\ell^2(m_1+m_2) & -\lambda m_2\ell^2 \\ -\lambda m_2\ell^2 & m_2 g\ell - \lambda m_2\ell^2 \end{vmatrix} = 0$$

$$\Rightarrow \left[(m_1+m_2)g\ell - \lambda\ell^2(m_1+m_2)\right]\left[m_2 g\ell - \lambda m_2\ell^2\right] - \lambda^2 m_2^2\ell^4 = 0 \quad \text{or}$$

$$(m_1+m_2)\ell\,[g-\lambda\ell]\,m_2\ell\,[g-\lambda\ell] - \lambda^2 m_2^2\ell^4 = 0$$

or $$(g-\lambda\ell)^2 = \lambda^2\frac{m_2}{m_1+m_2}\ell^2 \quad \text{or} \quad (g-\lambda\ell) = \pm\lambda\ell\sqrt{\frac{m_2}{m_1+m_2}}$$